$$\lim_{x \to 0} \frac{\sin x}{1 - \cos x}$$

In this problem attempt to evaluate:

$$\lim_{x \to 0} \frac{\sin x}{1 - \cos x}$$

using approximation.

a) Substitute linear approximations for $\sin x$ and $\cos x$ into this expression. Can you tell what happens in the limit?

b) Substitute quadratic approximations for $\sin x$ and $\cos x$ into this expression. Can you tell what happens in the limit?

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$$\lim_{x \to 0} \frac{\sin x}{1 - \cos x}$$

a) $\cos x \approx 1$, $\sin x \approx x$

$$\Rightarrow \lim_{x \to 0} \frac{x}{1 - 1} = \lim_{x \to 0} \frac{x}{0}$$

$\therefore$ It is hard to say what happens as $x \to 0$.

b) $\sin x \approx \sin(0) + \cos(0)x - \frac{\sin(0)x^2}{2} = x$

$\cos x \approx \cos(0) - \sin(0)x - \frac{\cos(0)x^2}{2} = 1 - 0 - \frac{1}{2}x^2 = 1 - \frac{1}{2}x^2$

$$\Rightarrow \lim_{x \to 0} \frac{x}{1 - 1 + \frac{1}{2}x^2} = \lim_{x \to 0} \frac{2x}{x^2} = \infty$$

$\therefore$ The denominator approaches 0 faster than the numerator and hence the expression goes to infinity as $x \to 0$.